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# Modified Method for One-Dimensional Cutting Stock Problem

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## To cite this article:

Niluka Rodrigo, WB Daundasekera, AAI Perera. Modified Method for One-Dimensional Cutting Stock Problem. *Software Engineering*. Vol. 3, No. 3, 2015, pp. 12-17. doi: 10.11648/j.se.20150303.11

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**Abstract:** Selection of feasible cutting patterns in order to minimize the rawmaterial wastage which is known as cutting stock problem has become a key factor of the success in today's competitive manufacturing industries. In this paper, solving a one-dimensional cutting stock problem is discussed. Our study is restricted to rawmaterial (main sheet) in a rectangular shape (different sizes), and cutting items are also considered as rectangular shape with known dimensions (assume that lengths of the main sheets and cutting items are equal). Pattern generation technique is used to nest the pieces of cutting items within the main sheet by minimizing rawmaterial wastage. A computer program using Matlab software package is developed to generate feasible patterns using the above algorithm for 1D cutting stock problem. Location of each feasible cutting pattern inside the main sheet is given in Cartesian Coordinate Plane. The *Branch and Bound* approach in solving integer programming problems is used to solve the problem.

**Keywords:** Cutting Stock Problem, *Branch and Bound Algorithm*, Pattern Generation, Matlab Software Package

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## 1. Introduction

Competence of production system becomes a key factor of the success in today's competitive manufacturing environment and industries. Productivity can be enhanced by minimizing waste, lead time and hence reducing the cost of manufacture. For that reason, Operations Research plays a major role in minimizing manufacture waste. Many people including scientists have contributed and engaged in their research and other activities to overcome above challenges. A cutting stock problem basically describes in two ways, One-dimensional (1D) and Two-dimensional (2D) cutting stock problems. It consists of cutting large pieces, which are available in stock with different shapes to produce smaller pieces which are known as items, in order to meet a given demand. This often results in smaller pieces which cannot be used for the production and considered as waste. So, cutting items should be designed to minimize waste of rawmaterials. Many researchers have worked on the cutting stock problem and developed different algorithms to solve the problem. Among them, Gilmore et al [1] conducted some of the earliest research in this area and one-dimensional cutting stock problem is solved using *Linear Programming Technique*. In this study, unlimited numbers of raw materials

with different lengths are assumed available in stock, and a mathematical model has been developed to minimize the total cutting cost of the stock length of the feasible cutting patterns and *Column Generation Algorithm* has been developed to generate feasible cutting patterns. Then, Gilmore has claimed that feasible cutting patterns are increased with the required cutting items and *Linear Programming Technique* is not applicable to solved mathematical model with too many variables. Gilmore [2] has made an approach for one-dimensional cutting stock problem as an extended of earliest paper (Gilmore et al (1961)) and cutting stock problem has been described as a NP-hard problem. A new and rapid algorithm for the knapsack problem and changes in the mathematical formulation<sup>1</sup> has been evolved and Gilmore has explained the procedure of the *Knapsack Method* using a test problem. Saad M.A Suliman [3] has modified *Branch and Bound Algorithm* to find feasible patterns for 1D cutting stock problem. In the case study, Saad has selected four different types of steel coils to cut from the standard steel coil with the 130 cm length and width of the main coil and widths of the required coils are equal. *Branch and Bound Algorithm* has

been explained using the example. Also, Sirirat and Peerayuth [4] has proposed a mathematical model with column generation technique by a *Branch and Bound procedure* and *Heuristic based on the First Fit decreasing method* for one-dimensional cutting stock problem. Sirirat has made assumptions that different lengths of items to be cut from a stock, Each item has associated a certain length, and each cutting pattern for a stock are not limited in the number of knives, but the sum of the length of items do not exceed a length of stock. These problems arise in industries such as garment, paper, glass etc. Beasley [5] has discussed the unconstrained two-dimensional cutting stock problem with guillotine cuts (cuts that start from one edge and parallel to the other two edges) and staged cuts (the cuts at the first stage are restricted to be guillotine cuts parallel to one axis, then the cuts at the second stage are restricted to be guillotine cuts parallel to the other axis and the cuts at the third stage are restricted to be guillotine cuts parallel to the original axis etc.).

There are different arrangements to cut required pieces from the existing raw material to maximize the used area and each arrangement is defined as a cutting pattern. Rodrigo et al [6] has presented modified *Branch and Bound Algorithm* and a computer program using Matlab software package to generate feasible cutting patterns. As a case study, 1200 cm long steel bars are considered to be cut into eight types (different lengths) of pieces according to the customer requirements. Rodrigo et al [7, 8] has presented modified *Branch and Bound Algorithm* for two-dimensional cutting stock problem with fixed size rawmaterial and a computer program using Matlab software package to generate feasible cutting patterns. Coromoto et al (2007) has used a *Parallel Algorithm* and *Sequential Algorithm* to solve the mathematical model which maximizes the total profit incurred by cutting  $n$  number of rectangular pieces from a

### 2.1. Mathematical Model (Gilmore and Gomory)

$$\begin{aligned} \text{Minimize } z &= \sum_{k=1}^r \sum_{j=1}^n c_{jk} x_{ijk} \quad (\text{Total loss}) \\ \text{Subject to } \sum_{k=1}^r \sum_{j=1}^n a_{ijk} x_{ijk} &\geq d_i \quad \text{for all } i = 1, 2, \dots, m \quad (\text{Demand Constraints}) \\ x_{ijk}, a_{ijk} &\geq 0 \quad \text{and Integer for all } k, i, j. \end{aligned}$$

The number of occurrences of the  $i^{\text{th}}$  piece in the  $j^{\text{th}}$  pattern of  $k^{\text{th}}$  rawmaterial should be found to find the optimum solution (minimum-waste arrangement) for the above mathematical model. Therefore, *Branch and Bound Algorithm* (Saad M.A Suliman, 2001) is used to generate feasible cutting patterns.

$$\text{Here, } \sum_{k=1}^r \sum_{i=1}^m a_{ijk} w_i \leq W \quad \text{for all } j = 1, 2, \dots, n,$$

where  $w_i$  is the width of the  $i^{\text{th}}$  item and  $W_k$  is the width of the  $k^{\text{th}}$  main sheet.

A computer program using Matlab software package is developed to generate feasible patterns using the above

large rectangular main sheet. Coromoto [9] observed that all cutting patterns can be obtained by means of horizontal and vertical builds of meta-rectangles and has used *Viswanathan and Bagchi Algorithm* to produce best horizontal and vertical builds.

In this study, modified *Branch and Bound Algorithm* is further modified to solve the one-dimensional (1D) cutting stock problem with different sizes of rawmaterials. Pattern generation technique is used to reduce the wastage and a computer program using Matlab software package is developed to generate feasible cutting patterns and to define the location of each item in each pattern within the Cartesian coordinate plane for one-dimensional rectangular shape cutting stock problem as an extended of the earliest paper (Rodrigo et al (2011)).

## 2. Materials and Methods

Any firm's main goal is to maximize the annual contribution periphery accruing from its manufacture and sales. By reducing wastages and maximizing sales, efficiency can be improved. Wastage can arises in many ways and at any step of production line cutting stock problem can be described under the rawmaterial wastage.

According to the selection, a mathematical model to minimize the rawmaterial wastage is formulated as follows:

- $m$  = Number of pieces,
- $n$  = Number of patterns,
- $r$  = Number of rawmaterials with different sizes,
- $a_{ijk}$  = The number of occurrences of the  $i^{\text{th}}$  piece in the  $j^{\text{th}}$  pattern of  $k^{\text{th}}$  rawmaterial,
- $x_{jk}$  = The number of main sheets being cut according to the  $j^{\text{th}}$  pattern of  $k^{\text{th}}$  rawmaterial,
- $c_{jk}$  = Cutting loss for each  $j^{\text{th}}$  pattern of  $k^{\text{th}}$  rawmaterial,
- $d_i$  = Demand of the  $i^{\text{th}}$  piece.

algorithm for 1D cutting stock problem. Consequently, generated feasible patterns are used to solve the mathematical model given in the paper to obtain the number of main sheets needed to satisfy the requirements of each piece and minimum cutting waste.

### 2.2. Modified Branch and Bound Algorithm

*Step 1:* Arrange required widths (or lengths) of rawmaterials,  $W_k$ ,  $k = 1, 2, \dots, r$  in decreasing order, ie  $W_1 > W_2 > \dots > W_r$ , where  $r$  = number of main sheets.

*Step 2:* Arrange required lengths,  $w_i$ ,  $i = 1, 2, \dots, m$  in decreasing order, ie  $w_1 > w_2 > \dots > w_m$ ,

where  $m$  = number of items.

Step 3: For  $i = 1, 2, \dots, m$ ;  $k = 1, 2, \dots, r$  and  $j = 1$  do Steps 4 to 6.

$$\text{Step 4: Set } a_{11k} = \left\lfloor \left[ \frac{W_k}{w_1} \right] \right\rfloor;$$

$$a_{ijk} = \left\lfloor \left[ \frac{\left( W_k - \sum_{z=1}^{i-1} a_{zjk} w_z \right)}{w_i} \right] \right\rfloor \quad (1)$$

where  $W_k$  is the length of the  $k^{\text{th}}$  main sheet. Here,  $a_{ijk}$  is the number of pieces of the  $i^{\text{th}}$  item in the  $j^{\text{th}}$  pattern along the width of the  $k^{\text{th}}$  main sheet and  $\lfloor [y] \rfloor$  is the greatest integer less than or equal to  $y$ .

$$\text{Step 5: Set } p_{ijk} = a_{ijk},$$

where  $p_{ijk}$  is the number of pieces of the  $i^{\text{th}}$  item in the  $j^{\text{th}}$  pattern of the  $k^{\text{th}}$  main sheet.

Step 6: Cut loss along the width of the  $k^{\text{th}}$  main sheet:

$$c_{jk} = \left( W_k - \sum_{i=1}^m a_{ijk} w_i \right)$$

Step 7: Set  $t = m - 1$ .

While  $t > 0$ , do Step 8.

Step 8: While  $a_{ijk} > 0$

set  $j = j + 1$  and do Step 9.

Step 9: Generate a new pattern according to the following conditions:

For  $z = 1, 2, \dots, t - 1$

$$\text{set } a_{zjk} = a_{z(j-1)k};$$

For  $z = t$

$$\text{set } a_{zjk} = a_{z(j-1)k} - 1;$$

For  $z = t+1, \dots, m$

calculate  $a_{zjk}$  using (1).

Go to Step 5.

Step 10: Set  $t = t - 1$ .

Step 11: STOP.

### 2.3. Case Study

Proposed algorithm to solve one-dimensional cutting stock problem with different sizes of rawmaterials is tested and analyzed to determine feasible and optimal cutting patterns. Following example will illustrate how to generate feasible cutting patterns by minimizing total cutting waste:

A paper mill uses paper rolls of width 1000 mm, 500 mm and 800 mm as rawmaterials to cut different sizes paper items according to the given specifications. The company has received an order according to the dimensions given in Table 1:

Table 1. Required item dimensions and demand.

Item No	1	2	3	4
Required widths (mm)	500	210	297	250
Demand (sheets)	1000	2500	1500	1500

## 3. Results and Discussion

Modified *Branch and Bound Algorithm* is applied to the above example to generate feasible cutting patterns as given below:

Step 1: For  $k = 1, 2, 3$  widths of rawmaterials  $W_k = 1000$  mm, 800 mm and 500 mm.

Step 2: For  $i = 1, 2, 3, 4$  widths of items  $w_i = 500$  mm, 297 mm, 250 mm and 210 mm.

Step 3: For  $i = 1, 2, \dots, 4$  and  $j = 1$  and  $k = 1$ , do Steps 4 to 6.

$$\text{Step 4: Set } a_{111} = \left\lfloor \left[ \frac{W_1}{w_1} \right] \right\rfloor = 2;$$

$$a_{211} = \left\lfloor \left[ \frac{\left[ W_1 - (w_1 a_{111}) \right]}{w_2} \right] \right\rfloor = 0;$$

$$a_{311} = \left\lfloor \left[ \frac{\left[ W_1 - (w_1 a_{111}) - (w_2 a_{211}) \right]}{w_3} \right] \right\rfloor = 0;$$

$$a_{411} = \left\lfloor \left[ \frac{\left[ W_1 - (w_1 a_{111}) - (w_2 a_{211}) - (w_3 a_{311}) \right]}{w_4} \right] \right\rfloor = 0;$$

Step 5: Set  $p_{ijk} = a_{ijk}$ ,

$$\text{Pattern 1} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Step 6: Total cut loss

$$(c_{11}) = \left( W_1 - \sum_{i=1}^m a_{i11} w_i \right) = 0 \text{ mm.}$$

Step 7: Set  $t = 4 - 1$ .

While  $t > 0$ , do Step 8.

Step 8:  $a_{311} = 0$

Step 8:  $a_{211} = 0$

Step 8:  $a_{111} > 0$

Step 9: Generate a new pattern according to the following conditions:

$$\text{set } a_{121} = a_{111} - 1 = 1$$

$$a_{221} = \left\lfloor \left[ \frac{\left[ W_1 - (w_1 a_{121}) \right]}{w_2} \right] \right\rfloor = 1;$$

$$a_{321} = \left\lceil \left[ \frac{W_1 - (w_1 a_{121}) - (w_2 a_{221})}{w_3} \right] \right\rceil = 0;$$

$$a_{421} = \left\lceil \left[ \frac{W_1 - (w_1 a_{121}) - (w_2 a_{221}) - (w_3 a_{321})}{w_4} \right] \right\rceil = 0;$$

Step 5: Set  $p_{ijk} = a_{ijk}$ ,

Pattern 2

Step 6: Total cut loss

$$(c_{21}) = \left( W_1 - \sum_{i=1}^m a_{i21} w_i \right) = 203 \text{ mm.}$$

The algorithm proceeds in the same manner to generate all the cutting patterns shown in Table 2.

Table 2. Generated cutting patterns.

Required widths	Cutting patterns (from 1000 mm rawmaterial)															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
500 mm	2	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
297 mm	0	1	0	0	0	3	2	2	1	1	1	0	0	0	0	0
250 mm	0	0	2	1	0	0	1	0	2	1	0	4	3	2	1	0
210 mm	0	0	0	1	2	0	0	1	0	2	3	0	1	2	3	4
Cut Loss (mm)	0	203	0	40	80	109	156	196	203	33	73	0	40	80	120	160
Required widths	Cutting patterns (from 800 mm rawmaterial)											(from 500 mm)				
	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
500 mm	1	1	1	0	0	0	0	0	0	0	1	0	0	0	0	
297 mm	1	0	0	2	1	1	1	0	0	0	0	1	0	0	0	
250 mm	0	1	0	0	2	1	0	3	2	1	0	0	0	2	1	
210 mm	0	0	1	0	0	1	2	0	1	2	3	0	0	0	1	
Cut Loss (mm)	3	50	90	206	3	43	83	50	90	130	170	0	203	0	40	80

# of main sheet = 3  
 Width of the main sheet = [1000, 800, 500]  
 Number of pieces = 4  
 Width of the pieces = [500,297,250,210]  
 pattern = (1)  
 Number of pieces of item (1) within (1)<sup>st</sup> sheet = (2)  
 Number of pieces of item (2) within (1)th sheet = (0)  
 Number of pieces of item (3) within (1)th sheet = (0)  
 Number of pieces of item (4) within (1)th sheet = (0)  
 Cutloss of pattern (1) = (0)  
 pattern = (2)  
 Number of pieces of item (1) within (1)th sheet = (1)  
 Number of pieces of item (2) within (1)th sheet = (1)  
 Number of pieces of item (3) within (1)th sheet = (0)  
 Number of pieces of item (4) within (1)th sheet = (0)  
 Cutloss of pattern (2) = (203)  
 pattern = (3)  
 Number of pieces of item (1) within (1)th sheet = (1)  
 Number of pieces of item (2) within (1)th sheet = (0)  
 Number of pieces of item (3) within (1)th sheet = (2)  
 Number of pieces of item (4) within (1)th sheet = (0)  
 Cutloss of pattern (3) = (0)  
 pattern = (4)  
 Number of pieces of item (1) within (1)th sheet = (1)  
 Number of pieces of item (2) within (1)th sheet = (0)  
 Number of pieces of item (3) within (1)th sheet = (1)  
 Number of pieces of item (4) within (1)th sheet = (1)  
 Cutloss of pattern (4) = (40)  
 pattern = (5)  
 Number of pieces of item (1) within (1)th sheet = (1)  
 Number of pieces of item (2) within (1)th sheet = (0)  
 Number of pieces of item (3) within (1)th sheet = (0)  
 Number of pieces of item (4) within (1)th sheet = (2)  
 Cutloss of pattern (5) = (80)

pattern = (6)  
 Number of pieces of item (1) within (1)th sheet = (0)  
 Number of pieces of item (2) within (1)th sheet = (3)  
 Number of pieces of item (3) within (1)th sheet = (0)  
 Number of pieces of item (4) within (1)th sheet = (0)  
 Cutloss of pattern (6) = (109)  
 pattern = (7)  
 Number of pieces of item (1) within (1)th sheet = (0)  
 Number of pieces of item (2) within (1)th sheet = (2)  
 Number of pieces of item (3) within (1)th sheet = (1)  
 Number of pieces of item (4) within (1)th sheet = (0)  
 Cutloss of pattern (7) = (156)  
 pattern = (8)  
 Number of pieces of item (1) within (1)th sheet = (0)  
 Number of pieces of item (2) within (1)th sheet = (2)  
 Number of pieces of item (3) within (1)th sheet = (0)  
 Number of pieces of item (4) within (1)th sheet = (1)  
 Cutloss of pattern (8) = (196)  
 pattern = (9)  
 Number of pieces of item (1) within (1)th sheet = (0)  
 Number of pieces of item (2) within (1)th sheet = (1)  
 Number of pieces of item (3) within (1)th sheet = (2)  
 Number of pieces of item (4) within (1)th sheet = (0)  
 Cutloss of pattern (9) = (203)  
 pattern = (10)  
 Number of pieces of item (1) within (1)th sheet = (0)  
 Number of pieces of item (2) within (1)th sheet = (1)  
 Number of pieces of item (3) within (1)th sheet = (1)  
 Number of pieces of item (4) within (1)th sheet = (2)  
 Cutloss of pattern (10) = (33)  
 pattern = (11)  
 Number of pieces of item (1) within (1)th sheet = (0)  
 Number of pieces of item (2) within (1)th sheet = (1)  
 Number of pieces of item (3) within (1)th sheet = (0)



Number of pieces of item (4) within (3)th sheet = (0)  
 Cutloss of pattern (30) = (0)  
 pattern = (31)  
 Number of pieces of item (1) within (3)th sheet = (0)  
 Number of pieces of item (2) within (3)th sheet = (0)  
 Number of pieces of item (3) within (3)th sheet = (1)  
 Number of pieces of item (4) within (3)th sheet = (1)  
 Cutloss of pattern (31) = (40)  
 pattern = (32)  
 Number of pieces of item (1) within (3)th sheet = (0)  
 Number of pieces of item (2) within (3)th sheet = (0)  
 Number of pieces of item (3) within (3)th sheet = (0)  
 Number of pieces of item (4) within (3)th sheet = (2)  
 Cutloss of pattern (32) = (80)  
 optimum  
 Number of patterns = (32)

There are 32 feasible cutting patterns available to cut rawmaterials with the dimensions 1000 mm, 800 mm and 500 mm into required items. The mathematical model is developed to design generated cutting patterns so that waste (cut loss) will be minimized and the optimum solution to the model is given in Table 3:

*Table 3. Optimum Solution.*

Required widths	Optimal Patterns			Demand
	12	17	31	
500 mm	0	1	0	1500
297 mm	0	1	0	1500
250 mm	4	0	1	3000
210 mm	0	0	1	500
# of sheets from each pattern	625	1500	500	

Cutting location:  
 Pattern = (12)  
 Number of pieces of item (1) within (1)th sheet = (0)  
 Number of pieces of item (2) within (1)th sheet = (0)  
 Number of pieces of item (3) within (1)th sheet = (4)  
 Number of pieces of item (4) within (1)th sheet = (0)  
 Cutloss of pattern (12) = (0)  
 Pattern = (17)  
 Number of pieces of item (1) within (2)th sheet = (1)  
 Number of pieces of item (2) within (2)th sheet = (1)  
 Number of pieces of item (3) within (2)th sheet = (0)  
 Number of pieces of item (4) within (2)th sheet = (0)  
 Cutloss of pattern (17) = (3)  
 Pattern = (31)  
 Number of pieces of item (1) within (3)th sheet = (0)  
 Number of pieces of item (2) within (3)th sheet = (0)  
 Number of pieces of item (3) within (3)th sheet = (1)  
 Number of pieces of item (4) within (3)th sheet = (1)  
 Cutloss of pattern (31) = (40)

### 4. Conclusion

In this study, a cutting stock problem is formulated as a

mathematical model based on the concept of cutting patterns. As given in Table 2, thirty two cutting patterns are generated and only three cutting patterns are selected as given in Table 3 to cut the main sheets according to the requirements. In this case study, the plant assumes that all the extra pieces from each item as wastage. Also, dimensions of cutting items are large and the total cut loss can be decreased if there are smaller cutting items. Identifying the position of each cutting item in each cutting pattern is also crucial to cut the main sheet. But, identification of cutting location is not fulfilled with previous studies. Advantage of this study over the previous studies is cutting locations of each piece of cutting items can be obtained by Cartesian coordinate system.

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